## Guide to the Graphing Workspace

## Using the Spreadsheet and Defining Functions

The columns in the spreadsheet may be filled in in one of three ways:
i) by hand
ii) using the "Recursive Definition Box"
iii) using the Commands cell for a column


Suppose we want to fill in column C1 so that it begins with 0 and increases by 1 s . We fill in the Recursive Definition Box as follows and then click the Set Column Values button.


The Beginning box is filled in with what we want the first entry in the row to be. The Next box is filled in with an equation that tells how to obtain each entry in the column from the preceding one. The equation in the Next box must have the form

Next=some expression that involves the word Previous
Suppose we want column 2 to begin with 5 and decreases by 3s. First, in the Choose Column drop-down box, choose C2. Then fill in the Beginning and Next boxes as follows and click the Set Column Values button.


Other examples of equations that might be used in the Next box include:

> Next $=2^{*}$ Previous
> Next $=4^{*}$ Previous +1
> Next=Previous ${ }^{\wedge} 2+3$

Important things to note:

1) The words Next and Previous must start with a capital letter and they must be spelled correctly.
2) Anytime you want to indicate multiplication in the Graphing Workspace, you must use an asterisk (*).

One way to use the Commands cells to fill in a column is to combine columns in some way. For example, type $=\mathrm{C} 1+\mathrm{C} 2$ in the Commands cell for column 3 and then click anywhere outside that cell.


## Note that a Commands cell must begin with $=$.

Another way to use a Commands cell to fill in a column is to use a previously defined function. Functions are defined in the Function Table, which is below the spreadsheet and above the graph.


Suppose we want to define the function $f x=5-3 x$. Type this in the Enter Function or Expression box and click the Define A Function button.


Note that you cannot just type 3 x . You must type $3^{*} \mathrm{x}$ to indicate 3 times x .
We can also use the Function Table box to evaluate expressions. Type $f(-4)$ in the Enter Function or Expression box and click the Evaluate Math Expression button.

| Enter Function or Expression: | f(-4) | Define A Function |  |
| :---: | :---: | :---: | :---: |
| Evaluated Expressions Appear Here: | 17 | Ev |  |
| Graph This Function: $f$ ¢ | From $\mathrm{x}=$ | Until $\mathrm{x}=$ | Graph Function |

We see that $f-4=17$.
While we are at it, let's define another function, $g x=2 x+1$. (Don't forget the asterisk! Also, make sure you click Define A Function after entering the function definition.)

Note that you must use different letters to refer to different functions. If we entered $\mathrm{f}(\mathrm{x})=2^{*} \mathrm{x}+1$, the previous definition for $f$ would be overwritten and $f$ would now only refer to the rule $2 x+1$. You can use any letter you want to refer to the input variable. You can use any letters except i and j to refer to functions. If you forget how one of your functions was defined or the recursive rule you used to fill in a column, check the box at the very bottom of the Graphing Workspace page. It keeps track of both recursive definitions and function definitions.

To use a previously defined function to fill in a column in the table, we do the following. Suppose we want to regard the entries in column 1 as our inputs. We want to fill in column 4 with the outputs that the function $f$ would produce for these inputs. (What would the first few outputs be?) In the Commands cell for column 4, type $=\mathrm{f}(\mathrm{C} 1)$. Then click anywhere outside this cell. CaluMath evaluates $f$ at each of the inputs in column 1 and puts the outputs in the corresponding rows of column 4.


Note that you cannot type $=\mathrm{f}$ or $=\mathrm{f}(\mathrm{x})$ or $=5-3^{*} \mathrm{x}$ or $\mathrm{f}(\mathrm{x})=5-3^{*} \mathrm{x}$ in the Commands cell for column 4. (You could type $=5-3 * C 1$.) You must tell CaluMath what column to use as the input column.

## Graphing Columns, Graphing Functions, Tracing Graphs

There will be times when we want to regard two columns in the spreadsheet as a set of ordered pairs and plot those ordered pairs as points on a graph. For example, suppose we want to view the entries in column 1 as representing the horizontal axis coordinates in a set of ordered pairs and the entries in column 2 as representing the vertical axis coordinates. In other words, the ordered pairs are ( 0,5 ), ( 1,2 ), ( $2,-1$ ), ( $3,-4$ ), etc. To plot these points, go the Graph Box above the spreadsheet and just below the Recursive Definition Box.


Do NOT enter anything in the From and Until boxes on the right-hand side. Make sure Horizontal is C1 and Vertical is C2 (because the values in column 1 are the horizontal axis coordinates and the values in column 2 are the vertical axis coordinates). Click the Points button. Scroll down and observe that some of the points are displayed on the graph. If we want to see all of the points, we need to rescale the axes of the graph.

Click on the Rescale Axes Menu button, which is below the large box of buttons underneath the graph.

| Selected Graph: | PointsGraph2 * | X Precision: 0 ¢ | Y Precision: 3 \% | Button Instructions |
| :---: | :---: | :---: | :---: | :---: |
| Highlight Point | Plot Point | Save This Point | Draw Segment | Draw Line |
| Draw Function Graph | End Function Graphs | Drag Graphs | End Dragging Graphs | Tangent |
| Draw Color: black $\dagger$ | Draw Anything | End Drawing | Remove Last Drawing | Graphic Image |
| Remove Graphs | End Graph Removal | Axes + Axes - | Equal Scaling | No Grid Lines |



After clicking the Rescale Axes Menu button, the first thing you should always do is click the Current Values button on the far right of the Rescale Axes Menu.


Generally, we will only want to change the first four values (Beginning and Ending X and Beginning and Ending Y). These values specify the displayed endpoints of the horizontal ( X ) and vertical ( Y ) axes.

Because the inputs in column 1 go from 0 to 20, let's specify a Beginning $X$ Value of -5 and an Ending $X$ Value of 25. Because the smallest value in column 2 is -55 , specify a Beginning Y Value of -60 . Click the Rescale Axes button on far right of the Rescale Axes menu.

Now look at the graph. Notice that no additional points were plotted. Click the Remove All Graphs button, which is just to the left of the Rescale Axes Menu button (which now says Hide Rescale Axes Menu). The plot of points should be removed. Now go back to the Graph Box above the spreadsheet, make sure Horizontal is still C1 and Vertical is still C2 and click the Points button again. Now all the ordered pairs of points from columns 1 and 2 in the spreadsheet are plotted.

Instead of just plotting points from the spreadsheet, let's graph the two functions we defined in the Function Table. Go to the Function Table, which is just above the graph, make sure that function $f$ is selected in the Graph This Function drop-down box, and click the Graph Function button.

## Hide Function Table



Notice that the formula for $f$ is displayed to the right of the graph. (Why does this graph go through the plot of points?)

Now choose function $g$ in the Graph This Function drop-down box and click the Graph Function button. The graph now looks like this:

Hide Function Table

| Enter Function or Expression: | $g(x)=2^{*} x+1$ |  |  |
| :---: | :---: | :---: | :---: |
| Evaluated Expressions Appear Here: | 9 | Evaluate Math Expression |  |
| Graph This Function: g * | From $\mathrm{x}=$ | Until $\mathrm{x}=$ | Graph Function |



PointsGraph2
$f(x)=5-3 * x$
$\mathrm{g}(\mathrm{x})=2^{*} \mathrm{x}+1$

The Graphing Workspace allows you to trace along the graph of a function. First, you must highlight a point on the graph. In the large box of buttons underneath the graph, click the Highlight Point button.


Read the instructions in the pop-up window that appears, then click Close. Choose function $g$ in the Select Graph box. Click Close to close the pop-up window. Now click on the graph of $g$. Notice that the coordinates of the point you highlighted are indicated to the right of the graph. Click the Drag Graphs button. Read the information in the pop-up window and click Close. Notice that the arrow turns into a hand when moved over the graph. Use the hand to drag the highlighted point along the graph. (You must keep holding the mouse button down while you drag the point.) Notice that the coordinates displayed on the right side of the graph change as you drag the point. Right now you can only drag the point to integer coordinates. This is because $X$ Precision is set to 0 .
$X$ Precision and $Y$ Precision boxes

| Selected Graph: | 9 t | X Precision: 0 , t | Y Precision: 3 , | Button Instructions |
| :---: | :---: | :---: | :---: | :---: |
| Highlight Point | Plot Point | Save This Point | Draw Segment | Draw Line |
| Draw Function Graph | End Function Graphs | Drag Graphs | End Dragging Graphs | Tangent |
| Draw Color black t | Draw Anything | End Drawing | Remove Last Drawing | Graphic Image |
| Remove Graphs | End Graph Removal | Axes + Axes - | Equal Scaling | No Grid Lines |

An X Precision of 0 means that $x$-values can only be integers (i.e., the 0 means no digits behind the decimal point). A Y Precision of 3 means that $y$-values can be displayed with up to 3 decimal places. Change $X$ Precision to 3 , then use the hand to move the point slightly along the line. Release the mouse button. Now grab the point again with the hand and slide it back and forth along the line. Notice how much more smoothly the point moves and that both the $x$ - and $y$-coordinates of the point are now displayed to 3 decimal places. Click the End Dragging Graphs button.

Suppose that we only want to display the graph of $g$ for $x$-values between 5 and 15 . In the Function Table, make sure $g$ is still selected in the Graph This Function dropdown box, put 5 in the From $\mathrm{x}=$ box and 15 in the Until $\mathrm{x}=$ box, and click the Graph Function button.



PointsGraph2 $\mathrm{g}(\mathrm{x})=2^{*} \mathrm{x}+1$
$\mathrm{f}(\mathrm{x})=5-3 * \mathrm{x}$

## Plotting Points, Drawing Segments/Lines, Dragging Graphs

This section assumes you are starting with a blank Graphing Workspace page. (You can just click Reload Page at the top of the Graphing Workspace page to get a fresh page.)

First, rescale the axes for the graph so that both the $x$ - and $y$-axes go from -10 to 10 . Remember to click Current Values after clicking on the Rescale Axes Menu button. After changing the beginning and ending $y$-values, click the Rescale Axes button.

We want to plot two points and draw a line through them. Suppose that the points we wish to plot are $(-5,-3)$ and $(4.1,7.6)$.

In the large box of buttons underneath the graph, click the Plot Point button. Click Close to close the pop-up window. Then experiment with clicking on the graph to plot points.


Notice two things. First, every time you plot a new point, the previous point is removed. Second, you can only plot points with integer x-coordinates, but ycoordinates are displayed to three decimal places. This is due to the current settings for X Precision and Y Precision.

Because the first point we want to plot has integer coordinates, change Y Precision to 0 . Then plot the point $(-5,-3)$. After plotting this point, click the Save This Point button.

The second point we want to plot is (4.1, 7.6). Change X Precision and Y Precision to 1. We could try to plot the point now, but very likely we would have to try several times before we click on the right location. There is another way. Click the View Options Menu button.

| Selected Graph: | $\square$ | X Precision: 0 * | Y Precision: 0 ¢ | Button Instructions |
| :---: | :---: | :---: | :---: | :---: |
| Highlight Point | Plot Point | Save This Point | Draw Segment | Draw Line) |
| Draw Function Graph | End Function Graphs | Drag Graphs | End Dragging Graphs | Tangent |
| Draw Color: black $\uparrow$ | Draw Anything | End Drawing | Remove Last Drawing | Graphic Image |
| Remove Graphs | End Graph Removal | Axes + Axes - | Equal Scaling | No Grid Lines |



The Options menu will be displayed underneath the Rescale Axes menu. On the lefthand side, underneath Prompt For Correct Point?, put a check mark next to Point. Then click the Change Options button. Click Close to close the pop-up window.


| Prompt For Correct Point? <br> (Check means yes) | Display Graph Equation (Check means yes) | Remove Previous Graph? <br> (Check means Yes) |
| :---: | :---: | :---: |
| $\square$ Tangent | $\square$ Tangent | $\checkmark$ Tangent |
| $\square$ Highlight | $\checkmark$ Highlight | $\checkmark$ Highlight |
| $\checkmark$ Point | $\checkmark$ Point | $\checkmark$ Point |

place check
mark here


Click the Plot Point button. (Because the Options have been changed, you must click to activate this button again.) Plot a point in the vicinity of (4.1, 7.6). Notice that a window appears asking you if you want to keep the point you clicked on or if you want to change its coordinates. If you happened to click on (4.1, 7.6), click the Keep button. Otherwise, enter $(4.1,7.6)$ in the box and click the Change button. Click the Save This Point button.

Now we can draw a line through our two points or a line segment joining them.


The Draw Line and Draw Segment buttons will draw a line or segment between the last two saved points. So if you want to use these features you must plan ahead and save the points you want to use to draw the line or segment. You cannot go back "after the fact" and draw a line or segment between two points that were plotted, but not saved. You will have to re-plot the points, saving each one after you plot it.

Click on the Draw Line or Draw Segment button. The Draw Line button produces the following. Note that the equation of the line through the two points is indicated to the right of the graph.


Saved1: (-5,-3)
Saved2: $(4.1,7.6)$
Line $1(x)=1.16483516^{*} x+2.82417582$

In addition to tracing along a graph, the Graphing Workspace also allows you to drag a graph. Click on the Drag Graphs button (in the middle of the box of buttons underneath the graph). Click Close to close the pop-up window. Use the hand that appears when the pointer is held over the graph and drag the line to a new location. Notice that the equation of the line changes as you drag it. Also note that the slope of the line is preserved (i.e., does not change) when dragging it. Click on the End Dragging Graphs button.

## Troubleshooting

1. 

 plot points anywhere on the axes and remove graphs. Click the Button Instructions button to read he details of these buttons.

Click the View Spreadsheet button below to display a spreadsheet that will allow you to enter ay d calculate values and graph table data. Once the spreadsheet is visible, click the Spreadsheet Instructions button to read the details of preadsheet operations.

You can resize the axes and change options regarding graphs using the buttons at the bottom of the page.


I typed =Previous+3 in the Next box and got this error message. You must type Next=Previous+3 in the Next box.
2.


Once the spreadsheet is visible, click the Spreadsheet Instructions button to read the details of spreadsheet operations.
You can resize the axes and change options regarding graphs using the buttons at the bottom of the page.


## Both the words Next and Previous must begin with a capital letter.

3. 



I forgot to use an asterisk (*) to indicate multiplication.
4.

$f(x)=2 * t+1$

I defined a function $\mathrm{f}(\mathrm{x})=2^{*} \mathrm{t}+1$ and clicked the Graph Function button, but no graph appears. I should have entered either $\mathrm{f}(\mathrm{t})=2^{*} \mathrm{t}+1$ or $\mathrm{f}(\mathrm{x})=2^{*} \mathrm{x}+1$.
5.


You can resize the axes and change options regarding graphs using the buttons at the bottom of the page.
View Spreadsheet


I omitted a parenthesis in the function definition.
6.


| Beginning X Value: | Ending X Value: | Begin | Ending Y Value: | Axes Width(in): | Axes Height(in): | Y Axis At $\mathrm{X}=$ : | $X$ Axis At $Y=$ : | Current Values |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -5 | 15 | -20 | 50 |  |  |  |  | Rescale Axes |



These boxes cannot be empty.


Remember to click the Current Values button immediately after clicking the Rescale Axes Menu button.
7.


I have my x-values set to go between 5 and 25, but CaluMath wants to place the $y$ axis at $x=0$ (because the value in the $Y$ Axis At $X=$ box is 0 ). I must either change the beginning $x$-value to be less than or equal to 0 or change the $Y$ Axis At $X=$ value
to be a number between 5 and 25 (inclusive). Here is what changing the Y Axis At X $=$ value to 5 does to the graph:



| Remove All Graphs | Hide Rescale Axes Menu View Options Menu Text + Text - |
| :--- | :--- | :--- |


| Beginning X Value: | Ending X Value: | ${ }^{\text {Beginning Y Yalue: }}$ | Ending Y Value: | Axes Width(in): | Axes Height(in): | Y Axis At $\mathrm{X}=$ : | X Axis At $\mathrm{Y}=$ : | Current Values |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 25 | -50 | 50 | 8 | 5 | 5 | 0 | Rescale Axes |

Note that the vertical axis of the graph now corresponds to the line $x=5$. Also note that the horizontal axis still corresponds to the line $y=0$ because the value in the $X$ Axis At $Y=$ box is 0 .
8.

If you think you have defined a function, but CaluMath does not seem to recognize it (e.g., you type $=\mathrm{f}(\mathrm{C} 1)$ in the Commands cell for column 2 and click outside that cell, but nothing is filled in in column 2), check to make sure you actually defined the function. Look in the box at the very bottom of the Graphing Workspace page to see if your function definition is recorded there. Another way to check is look in the Graph This Function drop-down box in the Function Table. If your function is not listed there, you did not define it. Make sure to click the Define A Function button after entering the formula for the function in the Enter Function or Expression box.


Function f does not appear here because I have not yet clicked the Define A Function button
9.

If you have defined multiple functions, make sure the function you want to graph is selected in the Graph This Function drop-down box in the Function Table.


Clicking the Graph Function button in the situation shown above will graph function $f$.
10.

If you want to highlight a point on a graph, make sure the graph you want to use is selected in the Selected Graph drop-down box.


Clicking the Highlight Point button will allow highlighting of points on the graph of function because that is the function in the Selected Graph box.

